BCT and Grue

January 20, 2021

Bayesian Confirmation Theory

What is it for some evidence E to provide some confirmation for a hypothesis H?

Bayesian Confirmation Theory.

Evidence E *confirms* hypothesis H just in case

$$\Pr(H \mid E) > \Pr(H) \tag{1}$$

We are using 'confirms' in a technical sense to mean something like "E is evidence for H" or "E supports H," etc.

Bayesian Confirmation Theory makes great use of *Bayes' Theorem*.

Bayes' Rule. Assume that $\Pr(E) > 0$. Then,

$$\begin{aligned} \Pr(H \mid E) &= \frac{\Pr(E \mid H)}{\Pr(E)} \cdot \Pr(H) && (2) \\ &= \frac{\Pr(E \mid H)}{\Pr(E \mid H) \cdot \Pr(H) + \Pr(E \mid \neg H) \cdot \Pr(\neg H)} \cdot \Pr(H) && (3) \end{aligned}$$

$$\Pr(X \mid Y) = \frac{\Pr(X \wedge Y)}{\Pr(Y)}, \text{ if } \Pr(Y) > 0$$

This allows us to calculate confirmatory support making use of information we might have available to us—e.g., the *likelihood* of that evidence according to each hypothesis plus your *priors* in those hypotheses.

Goodman's Grue

Consider the following properties:

$$x \text{ is } \textit{grue} \text{ iff} \begin{cases} x \text{ is green} & \text{if } x \text{ is observed before 2pm 1/20/21} \\ x \text{ is blue} & \text{if } x \text{ is not observed before 2pm 1/20/21} \end{cases}$$

$$x \text{ is } \textit{bleen} \text{ iff} \begin{cases} x \text{ is blue} & \text{if } x \text{ is observed before 2pm 1/20/21} \\ x \text{ is green} & \text{if } x \text{ is not observed before 2pm 1/20/21} \end{cases}$$

Suppose that you observe a green emerald. According to the Instance Principle, this confirms the hypothesis that *all emeralds are grue*.

But should it?

Suppose that there is an emerald over here, which we will all observe—for the first time ever—at exactly 2:01pm today. How do you think it'll look? Like this? Or like this?

Do you think you have good reason to answer as you do?

Do you think you have *better* reason than someone who disagreed with you?

Does BCT Help?

Let's break this question down into three parts. (1) Do our observations confirm the hypothesis that all emeralds are green? (2) Do our observations confirm the hypothesis that all emeralds are grue? (3) Do we have reason to think that our observations favor one of these hypotheses over the other? And, if so, on what grounds?

Let $\textsc{All Green}$ be the hypothesis that *all emeralds are green*.

Let $\textsc{All Grue}$ be the hypothesis that *all emeralds are grue*.

And let E_i say *the ith observed emerald looks like this*.

Our evidence so far, which we can write this way $\mathbf{E} = (E_1 \wedge E_2 \wedge \cdots \wedge E_n)$, is that all as of yet observed emeralds have looked like this.

1. **Do our observations confirm the hypothesis that all emeralds are green?**

   Let's check. Is the following true?

   $$\Pr(\textsc{All Green} \mid \mathbf{E}) > \Pr(\textsc{All Green})$$

   Given Bayes' Theorem, it is just in case:

   $$\frac{\Pr(\mathbf{E} \mid \textsc{All Green})}{\Pr(\mathbf{E})} > 1$$

   This is true because, given that $\textsc{All Green}$ entails $\mathbf{E}$, $\Pr(\mathbf{E} \mid \textsc{All Green}) = 1$, which is greater than $\Pr(\mathbf{E})$.

   So, good news! Our observations do confirm the hypothesis that all emeralds are green!

2. **Do our observations confirm the hypothesis that all emeralds are grue?**

   Let's check. Is the following true?

   $$\Pr(\textsc{All Grue} \mid \mathbf{E}) > \Pr(\textsc{All Grue})$$

   Given Bayes' Theorem, it is just in case:

   $$\frac{\Pr(\mathbf{E} \mid \textsc{All Grue})}{\Pr(\mathbf{E})} > 1$$

   But, because $\textsc{All Grue}$ also entails $\mathbf{E}$, it's also true that $\Pr(\mathbf{E} \mid \textsc{All Grue}) = 1$; and so the Grue hypothesis is also confirmed by our observations.

3. **Do we have reason to think that our observations favor one of these hypotheses over the other?**

   We haven't said much about what it is for some evidence to *better* support some hypothesis more than another. There are different ways of measuring conformational support. That said, because both hypotheses have the same *Bayesian Multiplier*:

   $$\Pr(\textsc{All Green} \mid \mathbf{E}) = \frac{1}{\Pr(\mathbf{E})} \cdot \Pr(\textsc{All Green})$$

   $$\Pr(\textsc{All Grue} \mid \mathbf{E}) = \frac{1}{\Pr(\mathbf{E})} \cdot \Pr(\textsc{All Grue})$$

   The only differences will come down to the *prior probabilities* assigned to the conflicting hypotheses.

   Lesson: Priors really matter!

Here are some ways of doing it.

$\Pr(\textsc{All Green} \mid \mathbf{E}) > \Pr(\textsc{All Grue} \mid \mathbf{E})$

$\Pr(\textsc{All Green} \mid \mathbf{E}) - \Pr(\textsc{All Green}) > \Pr(\textsc{All Grue} \mid \mathbf{E}) - \Pr(\textsc{All Grue})$

There are several other ways as well.